Propagation of Errors

In order to obtain the desired final result in an experiment it is usually necessary to add, subtract, multiply, and/or divide various measured quantities. It will be assumed that a probable fractional uncertainty has been assigned to each measured quantity (found by dividing the uncertainty by the most probable value of the quantity). If sufficient data have been taken to calculate an arithmetic mean (average) and find a distribution width, this fractional error will be the width divided by the arithmetic mean. If only one measurement of the quantity has been made, this fractional error will be the student's estimate of the probable uncertainty divided by this measurement. If sufficient effort has not been made to eliminate systematic errors, a probable uncertainty should be assigned to take them into account as well as random errors. It then becomes necessary to determine the way the probable uncertainties in the several different measured quantities combine to determine the probable uncertainty in the final result.

1. In addition, one adds the numerical values of the probable uncertainties in the quantities in order to determine the probable uncertainty in the sum. For example, suppose you need to add the 3 numbers N_1, N_2, N_3. Each of these has a uncertainty $\pm e_1$, $\pm e_2$, and $\pm e_3$, presumably much smaller than their relevant N-value, then you need the sum: $(N_1 \pm e_1) + (N_2 \pm e_2) + (N_3 \pm e_3)$. The maximum value obtainable would be $N_1+N_2+N_3+(e_1+e_2+e_3)$ and the minimum would be $N_1+N_2+N_3-(e_1+e_2+e_3)$. It is possible that + errors will cancel the - errors, giving no error in the sum. Since one cannot determine whether or not this is the case the rule for now is to be conservative and state the sum as $N_1 + N_2 + N_3 \pm (e_1 + e_2 + e_3)$.

2. In subtraction, one does likewise: $(N_1 \pm e_1) - (N_2 \pm e_2) = N_1 - N_2 \pm (e_1 + e_2)$

3. In multiplication, follow the same algebraic procedure as you would for multiplication of polynomials (remember $B \ll A$ and $D \ll C$): $(A \pm B) \times (C \pm D) = AC \pm BC \pm AD \pm BD$. Since BD is much smaller than either BC or AD, it can be neglected. If you do that, you can re-write the answer as: $AC \pm AC(B/A + D/C) = AC\,[1 \pm (B/A + D/C)]$ and note that B/A is the fractional error in A and D/C is the fractional error in C. It is clear then that you can simply add the fractional errors of each of the quantities and get the fractional error of the product. This technique of adding fractional errors can be extended to products of more than 2 numbers and also to quotients as shown below.

4. For the case of division: $(A \pm B)/(C \pm D) = A/C \pm (A/C)(B/A + D/C) = A/C \pm B/C \pm AD/C^2$. You can prove this by saying that the quotient must have the form $X \pm Y$ and cross-multiplying. The resulting equation is $A \pm B = (C \pm D)(X \pm Y) = CX \pm CY \pm DX \pm DY$. Now it's clear that $X = A/C$ because if the measurements were perfect, both B and D would be zero. Then solve for Y assuming D is small compared to C and can be dropped in the sum $C + D$.

In the examples below, let $A = 25 \pm 2$, $B = 5 \pm 1$, $C = 40 \pm 3$, $D = 20 \pm 1$, and $E = 30 \pm 5$ and evaluate:

1. $A + B$
2. AB
3. A/B
4. ABC
5. AB/CDE
6. $A + B - C + D - E$
7. $AB + C^2/A$
8. $\sqrt{AB}$

In order to do #8, just assume the answer has the form $X \pm Y$ and then square both sides. Solve for Y assuming $X = \sqrt{125}$.

PHYSICS 300 – – FALL 2004 – – LAB # 1

Forced, Damped Harmonic Oscillation

Introduction

The purpose of this experiment is to study the resonant properties of a driven, damped harmonic oscillator. This type of motion is characteristic of many physical phenomena. For example, radiation from atoms in a laser can be described with the same equations developed here...only the names of the variables are changed. We use a sheet of flat steel that experiences a force trying to straighten it when it is bent. Since this force is proportional to the amount of bending, it acts like a linear spring, so we will analyze the problem this way.

In this experiment a mirror is mounted on the end of a flat piece of springy steel and set into vibration as shown on the left side of Fig. 1. A beam of laser light is bounced off the mirror onto the wall, and when the mirror moves, so does the light spot on the wall. When the mirror vibrates, the light spot does also, and its motion provides information about the position of the mirror. It appears as a continuous line if the vibration frequency is high enough.

Question 1: Show that a beam of light incident on the mirror as shown on the left side of Fig. 1 will be deviated by reflection through an angle twice as large as the change of the mirror angle. That is, for the case shown, show that the reflected and incoming beam make an angle 2θ.

Question 2: How does the position of the spot on the wall depend on the position of the mirror? Clearly the angle changes as the spring vibrates, so the spot moves. But also the position of the mirror changes because it not only turns, but also moves. Make a careful analysis and determine the important parameters.

Theory

The right side of Fig. 1 shows the idealization of the linear oscillator we will use to analyze this experiment. Consider a mass M located at a distance x_1 from a fixed barrier connected to it with a stretched spring and held by another spring whose end can be driven at any frequency ω with peak-to-peak amplitude $2s$. When $s = 0$, the position of the right end of the right-hand spring is $x_1 + x_2$ as shown in Fig 1. The sum of the forces on the mass from the springs is $-k(x_1 - p + x) + k(x_2 - p - x)$, where k is the spring constant for both springs (assumed equal), p is their unstretched length, and x is the distance of the mass from its equilibrium point x_1. If x_1 and x_2 are equal, which is expected if the spring constants and unstretched lengths are the same, then the sum of the forces on the mass from the springs is simply $-2kx$. If the mass is moving, it experiences friction, and for simplicity we'll consider the case where the friction force is proportional to dx/dt, and so the sum of the forces on it is $-2kx - \beta(dx/dt)$ which must equal the mass times the acceleration, $M(d^2x/dt^2)$. The velocity-dependent friction force is $-\beta(dx/dt) = -\beta v$, where v is the velocity.

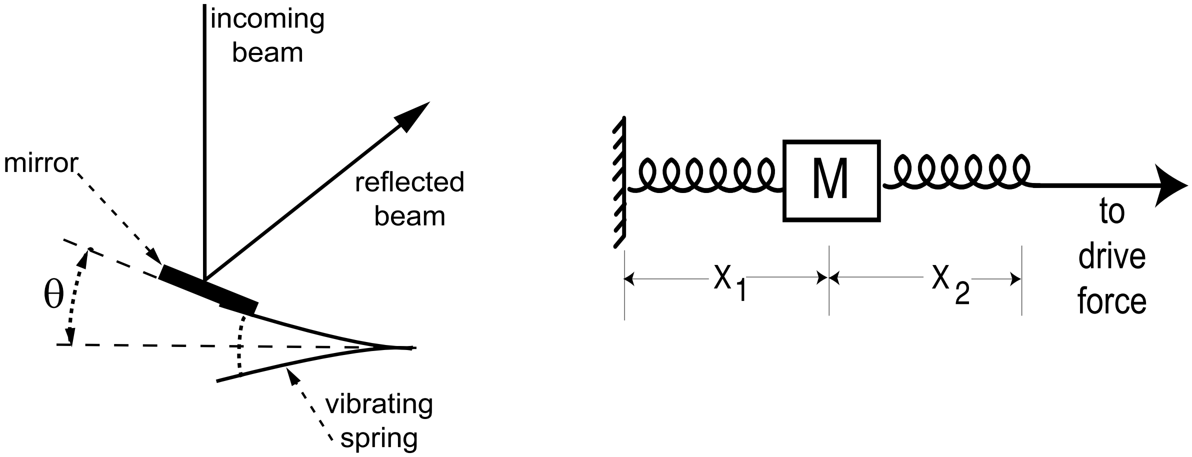

FIG. 1: The left side is a schematic diagram of the vibrating sheet metal spring and the light beam reflecting from the mirror mounted on it. The right side shows the idealization of this oscillator as a mass-spring oscillator. The mass is at equilibrium at position x_1 when it is at rest. Oscillations occur about x_1 at the driving frequency ω or, in the case of zero drive, at the resonant frequency ω_0.

Under these conditions, the motion of the mass when displaced from equilibrium by A is simply that of a damped oscillator, $x = A\cos\omega_0 t\, e^{-\gamma t/2}$ where $\omega_0 = \sqrt{K/M}, K = 2k$, and $\gamma = \beta/M$. Later we will discuss your measurement of this phenomenon. Now suppose that the right hand end of the right hand spring is vibrated so instead of the end being fixed at $x_1 + x_2$ its position is given by $x_1 + x_2 + s\sin\omega t$. Then the sum of the forces includes the driving force, and the equation of motion becomes

$$M\frac{d^2x}{dt^2} = -Kx - \beta\frac{dx}{dt} + F_0\sin\omega t \tag{1}$$

where $F_0 = Ks$. Equation 1 is the very famous damped, forced oscillator equation that reappears over and over in the physical sciences.

There are many possible solutions to this equation, but only those that correspond to physical reality are sought. Experimentally it is clear that the mass will oscillate at the driving frequency that can be varied over a wide range. The motion of the mass differs in phase from the drive even though its frequency is the same, but this will be hard to see in this experiment. A difference in phase means that the mass does not always move in the direction of the applied force, but may sometimes move in the opposite direction.

Try a solution of the equation in which the force is out of phase with the motion. For simplicity change the phase of the force instead of the phase of the motion (this clearly is the same as changing the phase of the motion by the negative of the force phase change) so $x = A\cos\omega t$ and then Eq. 1 will become

$$-MA\omega^2\cos\omega t = -AK\cos\omega t + \beta A\omega\sin\omega t + F_0\sin(\omega t - \phi) \tag{2}$$

where ϕ is the phase difference. This equation must hold for **all** values of time, so we can choose any time to evaluate it. At $t = 0$, and at $t = \pi/2\omega$, we find

$$MA\omega^2 = +AK + F_0\sin\phi \quad \text{and} \quad 0 = \beta A\omega + F_0\cos\phi \tag{3}$$

since $\sin(\pi/2 - \phi) = \cos\phi$. Solve the second part of Eq. 3 for $\cos\phi$ and calculate $\sin\phi$ using $\sin^2\phi = 1 - \cos^2\phi$ and substitute it into the first Eq. 3. Isolate the radical on one side of the equation and then square both sides. The result is $A^2(K - M\omega^2)^2 = F_0^2 - (\beta A\omega)^2$ from which

$$A = \frac{F_0/M}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}} \tag{4}$$

where the frequency $\omega_0 = \sqrt{K/M}$ is the oscillation frequency when there is zero driving force. This is called the natural, frequency of the oscillator, or the resonance frequency.

Question 3: Show that A in Eq. 4 has the dimension of distance. What is the required dimension of $\gamma = \beta/M$ for this to be true? Show that this is indeed the dimension of γ.

Question 4: In this derivation, sines and cosines were used in place of exponential notation, and the consequence was considerable extra algebra. Replace the $\sin(\omega t - \phi)$ term by $\Im(e^{i(\omega t - \phi)})$, replace the trial expression for x with $\Re(Ae^{i\omega t})$, separate the real and imaginary parts of the result, and solve for A. (Here $\Re$ and $\Im$ refer to real and imaginary parts respectively.) Compare your expression for A with Eq. 4.

Although Eq. 4 may look a little complicated, it actually has a very beautiful and simple form. The denominator clearly has a minimum when the driving frequency is equal to the resonance frequency, and this makes A have a maximum. If the energy damping rate $\gamma = \beta/M$ is very small (note that this is twice the amplitude damping rate where $x = A\cos\omega_0 t\, e^{-\gamma t/2}$), the maximum value of A becomes very large. The graph of A versus driving frequency is approximately symmetrical about the resonance frequency for small γ because of the square of the difference of the squares that appears under the radical.

You can also take Eq's. 3 above, solve them for the sine and cosine terms, and divide, thereby eliminating F_0. The result is, substituting ω_0^2 for K/M

$$-\tan\phi = (\omega^2 - \omega_0^2)M/\beta\omega \tag{5}$$

Question 5: Continuing your work on question 4, find ϕ using exponential notation.

Equations 4 and 5 describe the motion you are going to study in the laboratory. Notice that as the driving frequency approaches the resonance frequency from either above or below it, the amplitude of the oscillation A becomes very large. For $\omega = \omega_0$, $A = F_0/\beta\omega_0$ and diverges as $\beta \to 0$. If there were no friction the amplitude would become infinitely large, but before this happened the mirror would hit the driving magnet (see below) or spring would break. The amplitude of oscillation at a given damping is always near a maximum when the system is driven at its resonance frequency.

Also notice that if the driving frequency is very small, $\tan\phi$ is a very large positive number and therefore ϕ is approximately $\pm\pi/2$. Substitute $\phi = -\pi/2$ into Eq. 2 and find that the driving force is $F_0 \sin(\omega t + \pi/2)$ which is the same as $F_0 \cos\omega t$. The oscillation motion is almost exactly in phase with the driving force!!! Also notice that if the driving frequency is very much larger than the resonant frequency, then $\tan\phi$ becomes a very large negative number and this means $\phi \sim \pi/2$. Then the motion is almost exactly out of phase with the drive; when the spring pulls the mass moves away and when the spring pushes the mass moves against the push. This phenomenon is not readily observable with your oscillator, and so you need to do a separate, less precise experiment to observe it, as discussed below.

Procedure

In the first part of this experiment you will measure the damping time $1/\gamma$ by watching the decay of the oscillator using the laser spot on the wall. First, align the laser beam so it is pointing directly away from the wall and is incident nearly perpendicular to the mirror, but with enough of an angle so that the reflected beam misses the laser and hits the wall behind it. Set the spot on the wall at a convenient place for measurement. Then carefully displace the mirror by a few mm by bending the spring and releasing it. The spot on the wall should smear out into a line several cm long, and then slowly shrink back to a spot.

To determine γ you need to measure the time dependence of the length of this line, and since the decay time is only a few seconds, this is a bit difficult. Practice with your lab partner measuring this length after each second, and plot the results. Repeat the experiment several times. Plot your results on semi-log paper and use the slope to find γ, or use a least squares fit to $e^{-\gamma t}$. Be sure to propagate the uncertainties.

Question 6: Show that for an exponential decay, the measured decay constant is **independent** of the initial displacement of the mirror.

In the second part of this experiment, you will measure a resonance curve that should look like that of Fig. 2. This is just practice in getting used to the apparatus, because the work needed to answer question 7, a *required* part of this lab, will take a longer time. There is no known answer to that question, and so it's a matter of pure investigation for you to figure out. In this sense, it truly mimics real laboratory research.

The steel spring is magnetic, and to drive it we apply an oscillating magnetic field using an electronic oscillator whose frequency and amplitude can be varied. The circuit is exquisitely simple - just connect the magnet's leads to the oscillator's plug with the clip leads. Turn on the oscillator, set its frequency somewhere around 10 - 20 Hz, and adjust the amplitude so the laser spot on the wall is smeared into a line a few cm long. Then carefully maximize the length of this line by slowly varying the oscillator frequency. The line may stretch out very long, and you should reduce the drive amplitude until it is only a few cm long.

Now connect the output of the oscillator also to the frequency counter so you can determine the drive frequency simultaneously with observing the line on the wall. Then play with the frequency. Away from resonance the line will be short, only a few mm, and you'll have to measure this without changing the drive, so try to determine the best value for the amplitude.

Take several measurements of the oscillation amplitude at each of several driving frequencies. Be sure to wait for transients to die out (see below). You should be able to plot resonance curves like the one shown in Fig. 2. Be sure to show your error bars for both amplitude and frequency.

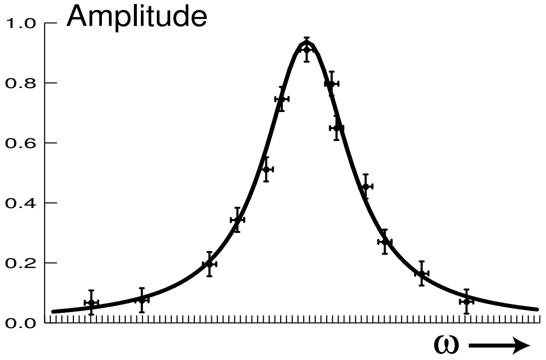

FIG. 2: The "measured" amplitudes are plotted against the driving frequency, and each data point has error bars in both dimensions. Each amplitude was measured several times, the results were averaged, and the uncertainty is the standard deviation from the average. The plotted values have all been divided by the measured maximum value. The (smaller) error bars on the frequency derive from estimates of the precision of reading the counter. The smooth curve was fitted to the data.

You can also determine γ from this measurement, and you should do so. Solve Eq. 4 for the case $A = A_{max}/2$ where $A_{max} = F_0/\omega_0\beta$ is the amplitude on resonance, $\omega = \omega_0$. There are two solutions, corresponding to the frequencies where the measured curve is at half of its maximum, and their difference is the full width at half maximum of the curve (FWHM is the standard abbreviation). Do the algebra to show that the FWHM = γ, determine this value from your measurements, and compare the result with that of the first part of the experiment. Be sure to propagate the uncertainties, and interpret your comparison of the two methods in the light of these values.

Now change something. The easiest thing to do is add a mass to the vibrating spring by simply putting on an alligator clip. Repeat the experiment, and observe what happens to the resonance frequency.

Question 7: Does the position of the alligator clip matter? Using the same alligator clip, take several resonance curves and plot the resonance frequency vs. position of the clip. You'll have to figure out some accurate way to measure this position. This problem has no simple solution - we don't know the answer. Only you will find it out. There is no "**right**" answer for this plot. Be sure to show your error bars.

For the third part of this experiment you need to observe the relation between the phase of oscillation and the driving frequency. To do this, simply hang a weight on a string about 1 m long to make a simple pendulum, and swing it with your hand. If you move VERY slowly, the weight will simply follow under you hand, in phase. If you oscillate your hand very fast, the weight will move in exactly the opposite direction, with an amplitude smaller than the motion of your hand. When you drive it at resonance, it will be clear that the motion is neither in phase nor out of phase, and careful observation will show that the phase is about halfway between these limits.

Hints and Kinks Department

The primary problem that you will face in this experiment arises from the transients of motion in the oscillation. The solutions to the equation derived earlier are valid only after a long period of time has elapsed from starting. They do not describe the motion at the beginning, nor do they account for any possible motion at any frequency other than the driving frequency. In class you have been told that if the oscillation has a component of motion at any frequency other than the driving frequency, then the total motion will be the sum of the motion at the driving frequency and the oscillation at the other frequency (call it ω'). This sum can be written as $A\cos\omega t + B\cos\omega' t$ which becomes, with the use of a trig identity,

$$2A\cos\left[(\omega+\omega')t/2\right]\cos\left[(\omega-\omega')t/2\right] + (B-A)\cos\omega' t \tag{6}$$

A plot of this motion shows one oscillation at the average of the frequencies contained in an envelope at the difference of the frequencies. For a while the oscillation will appear at close to the driving frequency but then the amplitude of the oscillation will decrease significantly. The laser spot will then wiggle a little bit and begin oscillating again. This peculiar behavior will persist until the oscillation at ω' has died away (i.e., $B = 0$ above). The loss of energy in the system at ω' depends on ω' and may take several seconds before becoming complete. If you try to measure the amplitude of an oscillation after a frequency change without waiting for the energy at the old frequency to be lost through friction, the results will be that the amplitude is not constant but varies with time. One way to tell if there is no energy left at any frequency other than the driving frequency is to observe the amplitude for 20 or 30 seconds. If it remains constant, you can take that measurement as the amplitude at that frequency.

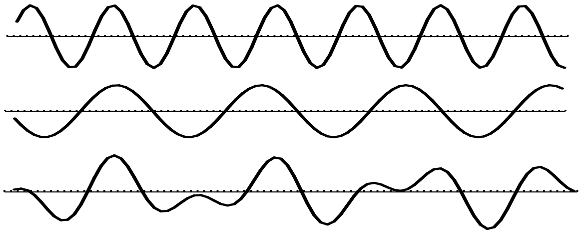

FIG. 3: The upper two curves show oscillations at quite different frequencies with approximately equal amplitudes. Their sum is plotted in the bottom curve on a different scale. It is clear that the superposition motion of the sum shows no simple characteristics, and that its amplitude varies with time.

PHYSICS 300 – – FALL 2003 – – LAB # 2

Coupled Oscillators

Introduction

In this experiment you are going to observe the normal modes of oscillation of several different mechanical systems, first on the air tracks and then using some coupled pendula. The normal modes of motion of a system of coupled oscillators are 'stable' with respect to time. That is, if you start the masses of a system oscillating in one of the normal modes and observe it for some time, the motion will have constant characteristics as its amplitude decays because of the ever present friction forces. Also, the frequency of oscillation of all the masses in the system is the same, and each of the masses executes simple harmonic motion at this frequency. The frequency is called the natural frequency of the normal mode or an eigenfrequency of the system. There are certain relationships between the eigenfrequencies and the frequencies of the uncoupled oscillators that will be discussed later.

If a system is excited into oscillations that are a mixture of normal modes, then the motion will change character over a period of time. Perhaps one of the masses will decrease amplitude and pass its energy to another mass, which will later pass the energy back. Such motion is clearly different from simple harmonic motion and therefore does not constitute a normal mode, even though the oscillations are all at the same frequency. The exchange of energy generally occurs at a frequency that is quite different from the oscillation frequency. (Read this paragraph a second time.)

Theory

Begin your study with two equal mass gliders on the air track as shown in Fig. 4. If the masses are not equal, certain complications arise that can obscure the simplicity of the motion, so you should remove all extra weights, clips, etc. and be sure that you have two sails of equal mass that you can mount on these gliders for damping the motion by air friction. The two gliders are coupled together with a spring as shown in the diagram. This coupling is somewhat tighter than the coupling you will later use with the two pendula, but it is not perfectly rigid; with a rigid bar the system would have only one mode of oscillation because the gliders would not be able to move relative to one another.

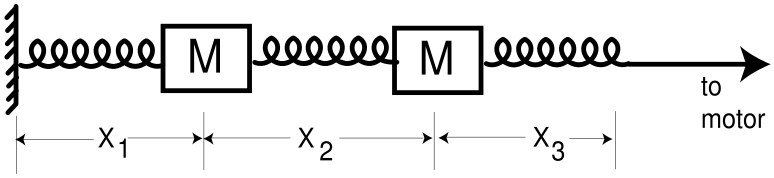

FIG. 4: The two masses and three spring constants are equal. At equilibrium, the three lengths x_i are also equal. The motor can only drive one end of one spring, but at resonance the energy is distributed among the masses.

If you displace the gliders symmetrically from their initial equilibrium positions and release them, their oscillations will die down in time, but the relative displacements of the two gliders will maintain a constant relationship. For example, if you push them toward one another by the same amount and release them, the displacement of one will always be equal in magnitude but opposite in direction to the displacement of the other. If you displace them both to one side by an equal amount and release them, they will oscillate together in a way that preserves their equilibrium separation. You should be able to observe both of these normal modes of oscillation with the pair of coupled masses before you go any further.

You should observe that the two normal modes described above have different frequencies. Measure these frequencies. You should rotate the sails on the gliders so that they are parallel to the air track to minimize damping in order to do this part of the experiment. There will then be many oscillations for you to count before all the energy of the system is lost. Now hold one of the masses fixed and measure the oscillation frequency of the other mass. Also, fix the second mass and measure the oscillation frequency of the first one. What is the relationship among the four frequencies you have measured?

In the case of one mass vibrating with the other held fixed the frequency is simply $\omega = \sqrt{2k/M}$ since the restoring force on the displaced mass is $2kx$ (because there are two springs each contributing kx). For the oscillations with the distance between the gliders constant, the frequency is $\omega = \sqrt{2k/2M}$ since there might just as well be a massless rigid bar between the gliders making it a mass of $2M$ oscillating under the same conditions as the single mass above. For the case of the masses oscillating in opposite directions, the frequency is $\omega = \sqrt{3k/M}$ since the restoring force from the middle spring is $2kx$ (its stretch is twice the displacement of either mass) plus the normal restoring force from the outside spring. The frequencies you measure should therefore be in the ratio of $1 : \sqrt{2} : \sqrt{3}$. Also notice that the sum of the squares of the individual oscillation frequencies (first one mass fixed and then the other) is equal to the sum of the squares of the frequencies of the normal modes. This is not an accident!

It is constructive to consider the motion of the center of mass (CM) of the system of gliders in each of the normal modes. In this case it is easy: in the high frequency mode (called the optical mode) the CM remains fixed at the middle of the center spring. You can hold the spring at this point without disturbing the oscillation. This point is called a node of the motion. In the low frequency normal mode (called acoustical mode) the CM oscillates at the acoustical frequency. In the optical mode there is motion with respect to the CM, but the CM is stationary. In the acoustical mode there is no motion with respect to the CM, but the CM moves. These are general properties of these two types of modes.

Consider the motion of the gliders with respect to the CM. How does the motion look from a coordinate system moving with the CM in each of the normal modes of oscillation? Describe it.

Procedure

The normal modes of motion represent the way a system of coupled masses oscillates naturally. In order to see what happens to a system of coupled oscillators driven at some frequency, consider the analogy to the case of a single mass where the motion is oscillatory at the driving frequency with an amplitude that is a maximum when the driving frequency is equal to the natural frequency. Since the coupled oscillators have two natural frequencies you might expect there to be two resonant maxima in the amplitude of the driven motion. In order to test this, you should mount the sails on the gliders for damping and proceed to drive them at various frequencies using the variable speed motor. You should record the amplitude of oscillation of each mass as a function of the driving frequency and plot the result. You should observe two resonant maxima for each mass as shown in Fig. 5, and the maxima should occur near the natural frequencies you have already measured.

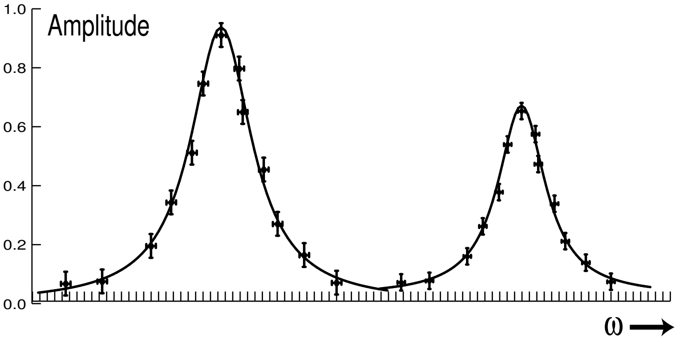

FIG. 5: The two resonances at the eigenfrequencies can be mapped out as shown. They are at different frequencies, and may have different heights and/or different widths.

Optional Advanced Work: If you would like to do some advanced work on this system you should go through exactly the same procedure for three coupled masses. There are now three normal modes. You might guess that one of them is an acoustical mode similar to that of two masses in which all three masses move together as if they were connected with rigid rods. This is a good guess but it's not correct simply because this middle mass would be expected to oscillate but the sum of the forces on it from the two springs would always be zero. This is because these springs retain their equilibrium length. It is therefore necessary that the middle mass oscillate with a larger amplitude but at approximately the same phase as the outside masses in the acoustical mode. This mode is hard to observe by starting the gliders oscillating at some point because you don't know the amplitude of the center glider. The frequency would be $\omega = \sqrt{2k/3M}$ as long as all three gliders have equal mass.

A little thought might lead you to guess that another normal mode consists of the two end masses oscillating in opposite directions with the center one perfectly stationary. You can verify this by starting the motion and then seeing if it maintains itself and if both the masses oscillate with the same frequency. Actually the center one can be said to oscillate at that frequency too, but with zero amplitude. If you do it carefully, the middle mass will never move as the oscillation of the end ones dies out. Show that the frequency of this oscillation is given by $\omega = \sqrt{2k/M}$.

The third normal mode is a little more difficult to discover by simply studying the equipment. You can find it by putting the sails on the gliders and driving the system at various frequencies. You can find the approximate frequency by using the rule of the sum of the squares of the frequencies described before. The oscillation frequency of each individual mass, if the the others were clamped, is $\omega = \sqrt{2k/M}$ (they're all equal) and so the sum of the squares is $6k/M$. The two frequencies we have discussed can be squared and summed to give $(8/3)k/M$ so the frequency of the third normal mode is expected to be $\omega = \sqrt{10k/3M}$. This is higher than the other two frequencies, but if is still within the range of your motors. You should measure the oscillation frequencies of the clamped masses to verify the above calculation, and when you find the normal mode, you should verify the sum of the squares relation.

Plot the amplitude of oscillation of one of the masses versus frequency and you will find *three* resonance maxima, two of which correspond to the normal modes we have already predicted. Observe the first normal mode and measure the amplitude and phase of the center glider. Observe the third normal mode and measure its frequency. If you set the drive at the resonance frequency of any of the modes and then switch off the motor, will the oscillations decay without changing frequency and still maintain the characteristics of the motion? If you are careful, you should be able to observe this.

Consider the motion of the CM of the three coupled oscillators. Describe the motion of the CM for each of the three normal modes of oscillation. Describe the motion of each of the gliders with respect to the CM for each of the three normal modes. Could you do this for motion of the system which is a mixture of normal modes?

Hints and Kinks Department

You must remember to wait some time for oscillations at the undesirable frequencies to die out after you change the driving frequency of the variable speed motor. Otherwise you will have the kind of problems of mixed modes described at the end of Experiment 1, namely, the motion will be a superposition of two or more frequencies. With coupled oscillators the system is much more complicated and too difficult to analyze in that simple way, so it's important to be patient.

The damping time of an air track glider with sail is also considerably longer than the spring in Experiment 1. You may want to measure it by clamping one glider and just watching the other one's oscillations damp out (of course, with no drive). Suffice it to say that the motion at any frequency which is not a pure eigenfrequency will decay into a superposition of the normal modes of the system resulting in a rather complicated motion.

It is very important to match the masses of the system as closely as possible. If you don't, the normal modes will not be symmetrical in the coordinate displacements. Of course normal modes exist for any system, whether or not the masses are equal. Usually the displacements are inversely proportional to the masses so that the CM of the system behaves in the same way as the center-of-mass of a system with equal masses. Remember that the sails do not have zero mass, so that the measurements you make of the resonant frequencies with other masses clamped should have the sails mounted but turned parallel to the track.

Be sure you do not drive the system into oscillations that are so large that the springs pop off. Be careful not to overheat the motors.

Coupled Oscillators With Variable Coupling

Connect the strings of two pendula with a massless (almost) rigid rod such as a soda straw. You can do it by cutting short slits in the ends of the straw and slipping the string of each pendulum into the slits at the end of the straw. The distance between the pendulum strings should be just a few mm less than the length of the straw. You should slide the coupling rod up the string until 9/10 or more of the string is below the coupling. Find and describe the normal modes of oscillation in the plane of the strings of this system of coupled oscillators (hint: there is an acoustical and an optical mode). What are the frequencies? Show that the measured values of the frequencies are consistent with the expected values. Do they also satisfy the sum of the squares rule? (How would you 'clamp' one of the masses in this case?)

You should observe and measure the frequencies of the normal modes for several different heights of this coupling bar. Make a plot of the frequencies of the normal modes versus height of the coupling bar. Is the result what you expect? Why or why not? Prove that the frequency of one of the normal modes is independent of the height of the bar, and that the other normal frequency depends on the square root of the distance from the bar to the pendulum bob. Which is the optical mode and which is the acoustical?

Put the bar near the top of the strings, displace one of the pendulum bobs, and release it. What happens? Describe the motion in detail. Go back and read the second paragraph of this write-up for a third time. How long does the phenomenon you observe take? This time depends on the degree of coupling and therefore on the height of the bar. Measure the time it takes versus height of the bar and make a graph. Does the curve look familiar? Is it related to anything we have done before? Is it related to the eigenfrequencies? How? Plot the time for energy transfer versus eigenfrequency to find out. Speculate on the formula for the time versus bar height graph. Can you derive it?

If you put the bar too low (lower than about 1/2 of the length of the strings) you will find that there is substantial coupling to another normal mode of oscillation, the torsional mode. Be careful to avoid it in the measurements above, but if you choose to do advanced work on this experiment you should study this mode as well as other normal modes of oscillation of the system. There are a total of four normal modes - why is this? Can you observe them all? Describe them.

For further advanced study you should try coupling three pendula by putting a slit in the middle of the straw and slipping the string of a third pendulum into it (see Fig. 6). what measurements can you make with this system? What can you predict about the results? Can you describe the motion of and with respect to the CM? Can you do it for just two pendula? Can you verify your predictions about the motion?

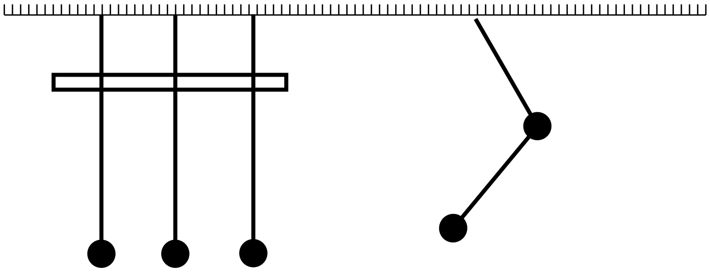

FIG. 6: The left side shows three coupled pendula, although you'll start this experiment using only two pendula. On the right is a "compound pendulum", which is simply another form of coupled oscillators.

You should also make a double pendulum, as shown on the right side of Fig. 6. What are the normal modes of motion? What are their frequencies? Make measurements. Do the frequencies obey the sum of the squares rule?

There are many options open to you in this experiment. Pick one or some of them that interest you and do them. Work carefully and accurately so that your efforts will not be wasted. It is better to do a small amount of work well than to try everything and do a sloppy job.

Hints and Kinks Department

Pick an amplitude that is large enough to be easily observable, but not so large that the pendulum weights collide. Remember also that a pendulum only executes simple harmonic motion if the maximum of excursion is a few degrees.

In both this and the air track experiments you should try to keep the oscillating masses from being disturbed by random air currents. Don't move around excessively, and don't breathe on the oscillators while you are trying to take measurements.